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The goal of the project was to explore the feasibility of novel configurations of heat pumping elements and heat exchangers for thermoacoustic heat engines, which have high potential for replacing refrigerators using polluting gases. The approach involved anisotropic systems, such as made possible by glass capillary array technology. This approach allows a heat exchange fluid to flow directly across the heat pumping element of the thermoacoustic heat engine (the "stack"), eliminating the necessity of separate heat exchangers and possibly improving efficiency. The research successfully determined both the theoretical and technological feasibility of the approach, and an actual prototype of an anisotropic stack/heat-exchanger unit was constructed. A part of the project involved the development of high power thermoacoustic refrigerators for testing the new systems. The research resulted in two patent disclosures.

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# ANISOTROPIC HEAT EXCHANGERS/STACK CONFIGURATIONS FOR THERMOACOUSTIC HEAT ENGINES

## Description of the project

This final report presents the accomplishments for ONR grant N00014-93-1-1127,"Anisotropic heat-exchangers/stack configurations for thermoacoustic heat engines". The goals of the project were to explore the theoretical and technological feasibility of novel component designs in a thermoacoustic heat engine. Such heat engines have high potential as replacements for current refrigerators which use polluting gases such as chlorofluorocarbons (CFC's), and as high power acoustic sources. The main scientific issue was the possible improvement in efficiency and simplification in fabrication resulting from the elimination of the separate heat exchangers at the ends of the heat pumping element (the "stack") of the thermoacoustic heat engine. Supplementary issues involved novel designs for testing components in high power thermoacoustic refrigerators.

## Description of the approaches

For the novel heat-exchanger/stack component, the approach was to use glass capillary array technology to form an anisotropic unit, allowing the oscillatory flow of the thermoacoustic fluid in one direction combined with the flow of a heat exchange fluid in a perpendicular direction. For the high power test-beds, the approach was to use high efficiency electric motors combined with other means of producing oscillatory motion, rather than using conventional reciprocating electromagnetic drives.

## Brief summary of accomplishments

The research showed that an anisotropic stack/heat-exchanger based on a glass capillary array was both theoretically and technologically feasible. The theoretical feasibility was determined with a consideration of the physics involved and with a computer calculation. The technological feasibility was determined through extensive literature searches on the properties of glass and other materials required in various fabrication schemes, and was ultimately demonstrated by the successful construction of a prototype of an anisotropic stack/heat-exchanger. A high efficiency, high power drive was also studied theoretically, and a prototype was constructed for experimental testing. A complete high power thermoacoustic refrigerator was constructed as a test bed for the novel components.

A physics graduate student, David Zhang, worked on the thermoacoustic refrigerator project for two years. He successfully completed the graduate course on thermoacoustic heat engines taught by Professor Steven L. Garrett. For the experimental part of the project, he significantly improved his laboratory, machine shop, and communication skills; the latter skill was crucial in the research, as it was necessary in order to obtain non-standard information about the glass capillaries, metal alloys, etc. required for the project. David should be commended for his successful fabrication of the first anisotropic stack/heat-exchanger, because it has involved techniques completely new to our laboratory.

A talk on this project was presented at the ONR meeting on the Environmentally Sound Ships Program held in Crystal City, MD, in March, 1995. Because of the patentable nature of this research, no public papers or talks were presented. Two patent applications have been submitted and are currently in process: "Stack/Heat-exchanger Unit for Thermoacoustic Heat Engines" (PSU Invention Disclosure No. 95-1500) and "High Power Oscillatory Drive" (PSU Invention Disclosure No. 93-1280, US Serial No. 08/610,790).

Because the technical parts of the research were not published in the open literature, copies of the patent disclosures are provided as appendices to supply technical information.

## Background and Scientific Issues

Before describing the accomplishments of our research, it would be worthwhile to provide some background information and terminology, and a discussion of scientific issues, relating to thermoacoustic heat engines.

A major contributor to the depletion of the earth's ozone layer is the reaction of the ozone with chlorofluorocarbons (CFC's) which are released into the earth's atmosphere from refrigerators which leak CFC's. In order to satisfy current and anticipated regulations governing the use of CFC's, it will be necessary to develop new types of refrigerators which do not use CFC's. A promising technology involves the thermoacoustic effect, in which the oscillatory motion of a gas in an acoustic field is coupled to a temperature gradient at a solid surface parallel to the motion. Reviews of this effect and its application in refrigerators and other heat engines (some nearing commercialization), are available in the literature.[1,2]

In order to increase the heat carrying capacity of the thermoacoustic heat engine, a large number of solid surfaces are used in a parallel configuration, as in a stack of plates, a spiraling sheet, or an array of capillaries or pins; this part of the thermoacoustic heat engine is referred to as the "stack". At each end of the stack are heat exchangers which would connect the refrigerator to an ambient temperature reservoir and to the load to be cooled. The sealed acoustic resonator is filled with a non-CFC gas, such as a helium-argon mixture. It should be noted that the spacing of the surfaces in the stack (or the inside diameter of capillaries in an array) is on the order of the thermal penetration depth for the gas, typically a few hundred microns.

Key elements in a high power thermoacoustic refrigerator are the heat exchangers at the ends of the stack. For an isothermal heat-exchanger, the length of the exchanger (in the direction of the gas particle velocity) should be on the order of the gas particle peak-to-peak displacement (as large as several millimeters). A difficulty arises from the disparity in the length scales between the stack (with a scale of several hundred microns) and the heat exchanger pipes (with a scale of several mm). The TAR could be improved if the heat-exchanger were incorporated into the stack with a matching length scale. This would form an anisotropic stack/heat-exchanger unit, which could transport large heat flows laterally (across the stack) but not longitudinally (along the stack). If such a heat-exchanger used a flowing fluid, rather than heat conduction, to transport the heat, then one could not only

handle higher heat loads, but one could also have graded exchanges with the external heat exchangers. That is, the temperature of the heat-exchanger fluid entering and exiting the external heat-exchanger could be made to match the temperature at the point of entry or exit of the exchanger in the stack.

### Theoretical Studies of the Feasibility of an Anisotropic Stack/Heat-Exchanger

The elements of our proposed anisotropic stack/heat-exchanger system are presented in the patent disclosure in Appendix A. Briefly, the system uses capillaries to form a stack, but the capillaries have sections with reduced outer diameters at each end. When the capillaries are bonded together to form the stack, the reduced diameters form narrow slits through which heat-exchanger fluid may flow. The theoretical question to be addressed was whether having to push the heat-exchanger fluid through narrow slits would be effective or detrimental. After spending some time studying heat-exchanger texts, the student was able to show that the narrow slit system should work well. An outline of the calculation may be found in Appendix A.

### The Test Prototype Anisotropic Stack/Heat-exchanger Unit

The anisotropic stack/heat-exchanger unit which we have fabricated is illustrated in Fig. 1c. The stack is composed of 50 mm long, 600 $\mu$m diameter glass capillaries, with $\sim$50 $\mu$m wall thickness, fused into a square array. The capillaries have flattened sections near each end, forming an array of 2 mm long, $\sim$200 $\mu$m wide slits passing through the stack. The thermoacoustic fluid oscillates inside the capillary pores, and a heat-exchanger fluid flows at right angles through the narrow slits between the capillaries. The heat to or from the thermoacoustic fluid near the ends of the stack conducts through the thin walls of the capillaries, and is transported by the heat-exchange fluid.

### The Method of Fabrication for the Anisotropic Stack/Heat-exchanger Unit

Originally, the method for fabricating the anisotropic stack/heat-exchanger unit was envisioned as drawing a clad optical fiber so as to have precisely positioned necked-down regions, cutting these into sections, and bonding the sections into an array analogous to Fig. 1c. A final step would be to leach out the inner core of the clad optical fiber. However, during the past year a different method, which uses only commercial straight glass capillary, was conceived, as follows.

The current method for fabricating the anisotropic stack/heat-exchanger unit is illustrated in Fig. 1a through 1c. The prototype unit is formed in a frame with inside dimensions of 10 x 15 x 50 mm. 2 mm wide slots, shown with protruding strips in Figs. 1a and 1b, are positioned where the array of heat-exchanger slits will be located. The fabrication procedure is to place one layer of glass capillary sections longitudinally (along the 50 mm dimension) in the frame, followed by a flat metal strip (2 mm width and 100 $\mu$m thick) across the capillaries, passing through each set of slots at the ends of the frame. Layers of the glass capillaries and metal strips are added alternately to fill the frame.

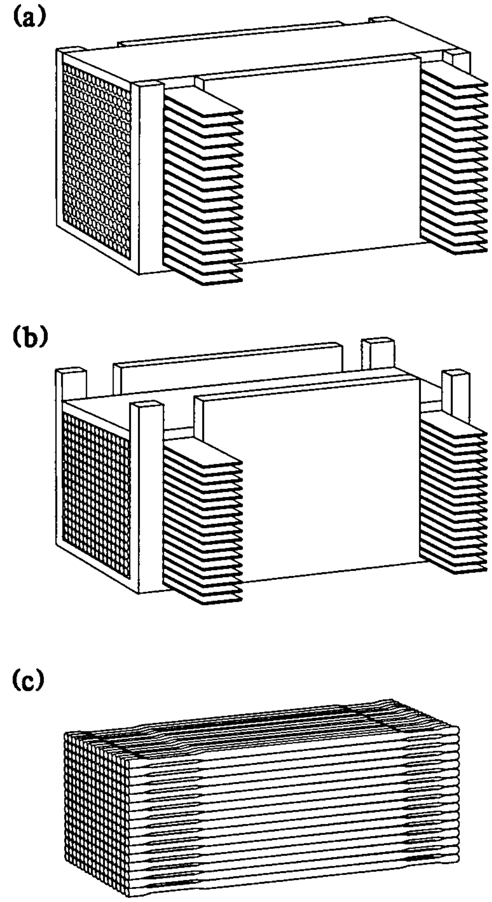


Fig. 1. An illustration of the method for fabricating the anisotropic stack/heat-exchanger unit using glass capillaries.

It should be noted that at this point the glass capillaries are not close-packed, and the cross-section of the array is not square, because the layers of capillaries are separated by the thickness of the metal strips, as shown in Fig. 1a. This configuration of capillaries and metal strips is covered with a metal plate which is lightly spring loaded so as to compress the capillary and metal strip array. The entire assembly is placed on a slowly rotating shaft inside a furnace and heated to slightly below the softening temperature of the glass (~800 C). The heating is done in an inert gas atmosphere to avoid excessive oxidation of the metal surfaces. With gradual heating, the capillary array eventually flows into the configuration shown in Fig. 1b., where the glass capillaries now form a square close-packed array, and in the region of the metal strips the circular cross-section of each capillary deforms into a rectangle. A simple calculation based on a perimeter preserving deformation shows that the metal strips may occupy as much as 2 - $\pi/2$ or ~43% of the height of the array. The next step in the process is to place the assembly in acid and remove all of the metal, leaving the glass capillary array, with open slits for the heat exchange fluid, as shown in Fig. 1c. The final step is to epoxy the unit into a manifold so that the unit could be mounted into an acoustic resonator and heat-exchanger fluid connections could be made.

The fabrication method outlined above required considerable prior study, particularly in the selection of materials. For example, readily available quartz capillary could not be used because its softening temperature (~1600 C) was so high that it severely limited the metals which could be used for the frame and strips; most metals would melt before the quartz softened. Borosilicate (Corning glass code 7740) glass, which softens at ~800 C, was necessary, and with considerable effort, a company which sold thin-walled borosilicate glass capillary was located.

Metal removal techniques often use aluminum, which dissolves in NaOH, but aluminum melts at 649 C, and hence could not be used even with the borosilicate glass. Many high melting temperature metals are not readily dissolved, or are not sufficiently stiff to remain flat when used for the metal strips crossing the capillary array. After a considerable search, an "invar" alloy was selected, so as to match the thermal expansion of the glass during the fusing process.

## Development of High Power Test Resonators

In order to test the new stack/heat-exchanger system at high powers, an acoustic resonator with a high amplitude drive was constructed. The resonator is 2.5 cm in diameter and 70 cm in length. The drive is a model aircraft engine, capable of rotating at 20000 RPM (over 300 Hz), with a diameter matching the resonator and a stroke of 2.5 cm. The engine was modified so that the intake and exhaust ports are sealed. A problem was how to drive the shaft of the engine, since a powerful high-speed motor, with regulated speed, was required. After considerable searching, it was found that a readily available woodworking router was ideally suited. Such routers may have speeds variable from 8000 to 24000 RPM, with a shaft transducer and feedback system which maintains constant speed under varying loads. The graduate student has tested the acoustic resonator and drive, and has measured a resonant frequency of 240 Hz and an acoustic pressure amplitude of 0.8 atm, consistent with the particle velocity produced by the drive.

A significant contribution by Steven L. Garrett has been the idea of using an annular geometry for the acoustic resonator in a TAR. In this case, a piston acoustic drive is not used, but instead a rigid wall is placed across the annular waveguide, and the entire resonator is oscillated in a torsional manner. A problem was how to drive such a TAR. Our solution was to use the high efficiency of an electric motor together with a scheme using powerful permanent magnets to convert the rotary motion of the motor to torsional oscillations. The technical details of the scheme are presented in Appendix B.

## References


1. G. W. Swift, J. Acoust. Soc. Am. **84**, 1145 (1988). Thermoacoustic engines.

2. G. W. Swift, *Physics Today* , July 1995, p. 22.

# APPENDIX A

## Invention Disclosure for a Stack/Heat-exchanger Unit for Thermoacoustic Heat Engines

A major contributor to the depletion of the earth's ozone layer is the reaction of the ozone with chlorofluorocarbons (CFC's) which are released into the earth's atmosphere from refrigerators which leak CFC's. [Note: by physics definition, refrigerators include air conditioners, etc.] In order to satisfy current and anticipated regulations governing the use of CFC's, it will be necessary to develop new types of refrigerators which do not use CFC's. A promising technology involves the thermoacoustic effect, in which the oscillatory motion of a gas in an acoustic field is coupled to a temperature gradient at a solid surface parallel to the motion. Reviews of this effect and its application in refrigerators and other heat engines (some nearing commercialization), are available in the literature.[1,2]

In order to increase the heat carrying capacity of the thermoacoustic heat engine, a large number of solid surfaces are used in a parallel configuration, as in a stack of plates, a spiraling sheet, or an array of capillaries; this part of the thermoacoustic heat engine is referred to as the "stack". An exploded view of a thermoacoustic refrigerator (TAR) is shown in Fig. 1. At the left in Fig. 1 is the acoustic driver, represented as an oscillating piston. Next is an open section of a longitudinal standing wave acoustic resonator, followed by the stack. At each end of the stack are heat exchangers, represented by fluid-carrying pipes in Fig. 1, which would connect the refrigerator to an ambient temperature reservoir and to the load to be cooled. Finally, at the right end in Fig. 1 is the end cap for the acoustic resonator. The sealed acoustic resonator is filled with a non-CFC gas, such as a helium-argon mixture. It should be noted that the spacing of the surfaces in the stack (or the inside diameter of capillaries in an array) is on the order of the thermal penetration depth for the gas, typically a few hundred microns.

Key elements in a high power thermoacoustic refrigerator are the heat exchangers at the ends of the stack, illustrated in Fig. 2. For an isothermal heat-exchanger, the length of the exchanger (in the direction of the gas particle velocity) should be on the order of the gas particle peak-to-peak displacement (as large as several millimeters). Such an isothermal exchanger poses a fundamental limitation, because the isothermal surface (unlike the surface with a temperature gradient in the stack) is a source of acoustic power loss. Another difficulty arises from the disparity in the length scales between the stack (with a scale of several hundred microns) and the heat exchanger pipes (with a scale of several mm). The TAR could be improved if the heat-exchanger were incorporated into the stack with a matching length scale. This would form an anisotropic stack/heat-exchanger unit, which could transport large heat flows laterally (across the stack) but not longitudinally (along the stack). If such a heat-exchanger used a flowing fluid, rather than heat conduction, to transport the heat, then one could not only handle higher heat loads, but one could also have graded exchanges with the external heat exchangers. That is, the temperature of the heat-exchanger fluid entering and exiting the external heat-exchanger could be made to match the temperature at the point of entry or exit of the exchanger in the stack.

A possible anisotropic stack/heat-exchanger unit is illustrated in Fig. 3; for simplicity the unit has been drawn with a square cross section, and lengths are not drawn to scale. The unit is composed of an array of capillaries (thin-walled, a few hundred microns in diameter), as in a conventional stack of this type, but now the capillaries have a reduced outer diameter near each end, as illustrated by the single capillary in Fig. 3. In the stack/heat-exchanger unit, the capillaries are bonded together so that the regions having the full diameter are completely sealed, while the regions of reduced diameter form narrow ($\sim$100 $\mu$m) open slits. The acoustic gas oscillates inside the capillary pores, and a heat-exchanger fluid flows at right angles through the narrow slits between the capillaries. The heat to or from the acoustic gas near the ends of the stack conducts through the thin walls of the capillaries, and is transported by the heat-exchange fluid. A question which arises is: Will having to push the heat-exchange fluid through narrow slits be detrimental? To address this question, some heat-exchanger fundamentals should be reviewed.

Some basic concepts for heat-exchangers are illustrated in Fig. 4. We consider a geometry consisting of two parallel isothermal walls at temperature $T_w$ with a heat-exchange fluid flowing between them, entering at a temperature $T_0 < T_w$. Heat leaving the plates is carried away by the moving heat-exchange fluid. Two basic modes for heat-exchange fluid flow, laminar and turbulent, are illustrated in Figs. 4a and 4b. The flow profile for laminar flow is shown by the parabola-shaped curves in Fig. 4a; the flow is parallel to the walls, is maximum in the center, and drops to zero at the walls. At the entrance (the left end in Fig. 4a) the temperature rises abruptly at the walls. For a wall spacing typical of refrigeration tubing (several mm), the consequence of these profiles is that the heat diffuses only a small distance from the walls where the flow is small, so that little heat is carried away. If the fluid is pushed fast enough between the walls, the flow becomes turbulent, as shown in Fig. 4b. In this case, perpendicular flow of the fluid near the walls convects the heat into the center of the channel where it is more effectively carried away by the mean flow of the fluid. For typical millimeter sized refrigeration tubing, turbulent flow is better for heat exchange. We now use a theoretical calculation to see if making the walls close together (as in narrow slits) might make the laminar flow situation more effective.

The theory involves the equation of continuity, the Navier-Stokes equation, and conservation of energy, including heat conduction:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \tag{1}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} P + \eta \nabla^2 \vec{v} + \left(\xi + \frac{1}{3}\eta\right) \vec{\nabla}\left(\vec{\nabla} \cdot \vec{v}\right) \tag{2}$$

$$\frac{\partial S}{\partial t} + \vec{v} \cdot \vec{\nabla} S = \vec{\nabla} \cdot \left(\kappa \vec{\nabla} T\right) \tag{3}$$

where, for the heat-exchange fluid, $\rho$ is the mass density, $\vec{v}$ is the flow velocity field, $P$ is the pressure, $\eta$ and $\xi$ are the shear and bulk viscosities, $\kappa$ is the thermal conductivity, $T$ is temperature, and $S$ is the entropy.

For steady, incompressible flow between parallel walls, with spacing $2y_0$ as illustrated in Fig. 4c, the first two equations yield:

$$P(x,y) = -\frac{\Delta P}{L}x \tag{4}$$

$$\vec{v}(x,y) = \frac{\Delta P y_0^2}{2\eta L}\left(1 - \frac{y^2}{y_0^2}\right) \tag{5}$$

where $L$ is the length from the left to the right end of the heat-exchanger and $\Delta P$ is the pressure drop across $L$. The net flow of heat exchange fluid is proportional to $y_0^2$, so that it would seem that narrow slits would be a disadvantage. Indeed, this impedance limits the flow to laminar for reasonable values of $\Delta P$. However, the calculation must be continued to find the the net heat transport, given by the thermal conductance:

$$\frac{\dot{Q}}{\Delta T} = \left(\frac{\Delta P \rho C_p}{8\eta L}\right)\sum_{n=1}^{\infty} A_n y_0^2\left[1 - e^{-(\zeta_n/y_0)^4}\right] \tag{6}$$

where

$$\zeta_n = \left[\frac{2\gamma_n \eta L^2 \kappa}{\Delta P \rho C_p}\right]^{1/4} \tag{7}$$

$\Delta T = T_w - T_0$, and $\gamma_n$ is an eigenvalue.

For large $y_0$ (3 mm), $\dot{Q}/\Delta T$ decreases with $y_0$ , reiterating that laminar flow is undesirable. However, for small $y_0$ it increases. For $L \sim 5$ - 10 cm, the maximum thermal conductance occurs for a wall spacing of a hundred microns, the same as the typical spacing for a stack. This favorable result may be seen as arising from the similarity between the formula for the thermal penetration depth

$$\delta = \sqrt{\frac{\kappa}{\rho C_p f}} \tag{8}$$

and that for the optimum plate spacing

$$2y_0 = \sqrt{\frac{\kappa}{\rho C_p (v/L)}} \tag{9}$$

where $f$ is the frequency of the acoustic resonance and $v$ is the average heat-exchanger fluid flow velocity. For a typical TAR, $f$ and $v/L$ are comparable.

Another quantity to consider is the energy lost through viscosity in pushing the heat-exchange fluid through the narrow slits. This is calculated with the product of $P$ and $v$. If one uses typical values for the parameters for a stack, one finds that the performance of a stack/heat-exchanger unit as illustrated in Fig. 3, with 100 $\mu$m slits, is comparable to that of a conventional TAR heat exchanger formed with 3 mm ID copper refrigerator tubing, but with the advantages that there is much better thermal contact with the thermoacoustic gas in the capillaries, and by using several sections of slits, a graded heat exchanger may be formed. Furthermore, a glass or plastic capillary array incorporated into the heat-exchanger or stack has the advantages of being mechanically (and chemically) robust, easily cleaned, and having high porosities.

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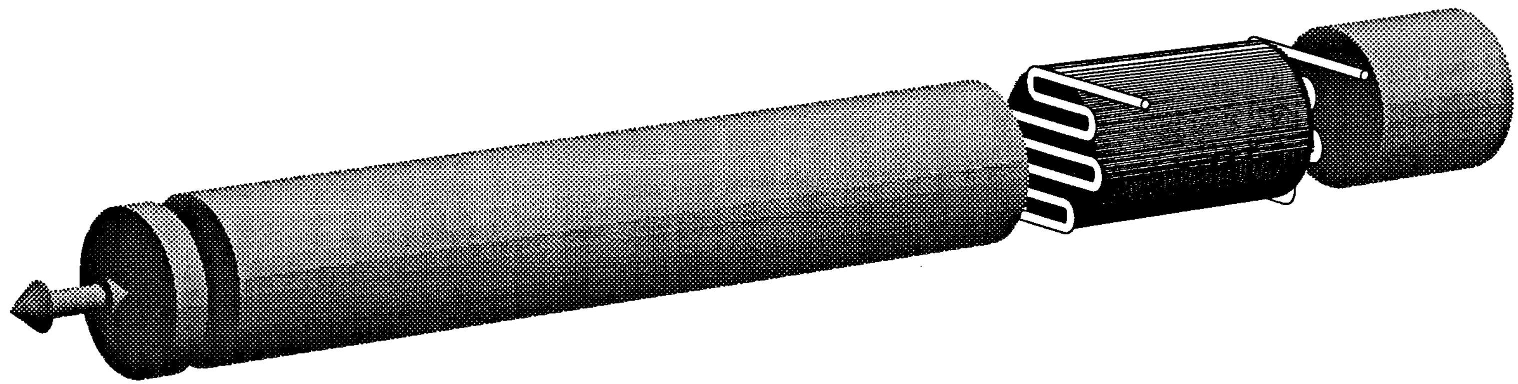


Fig. 1

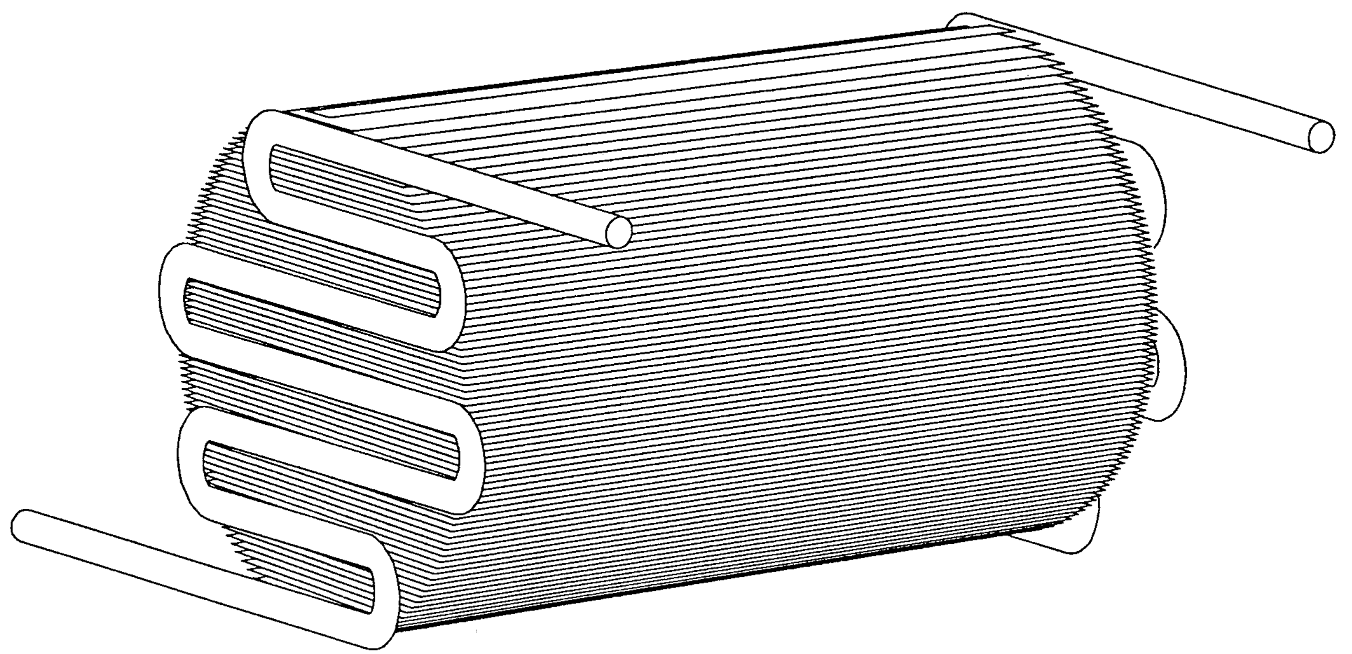


Fig. 2

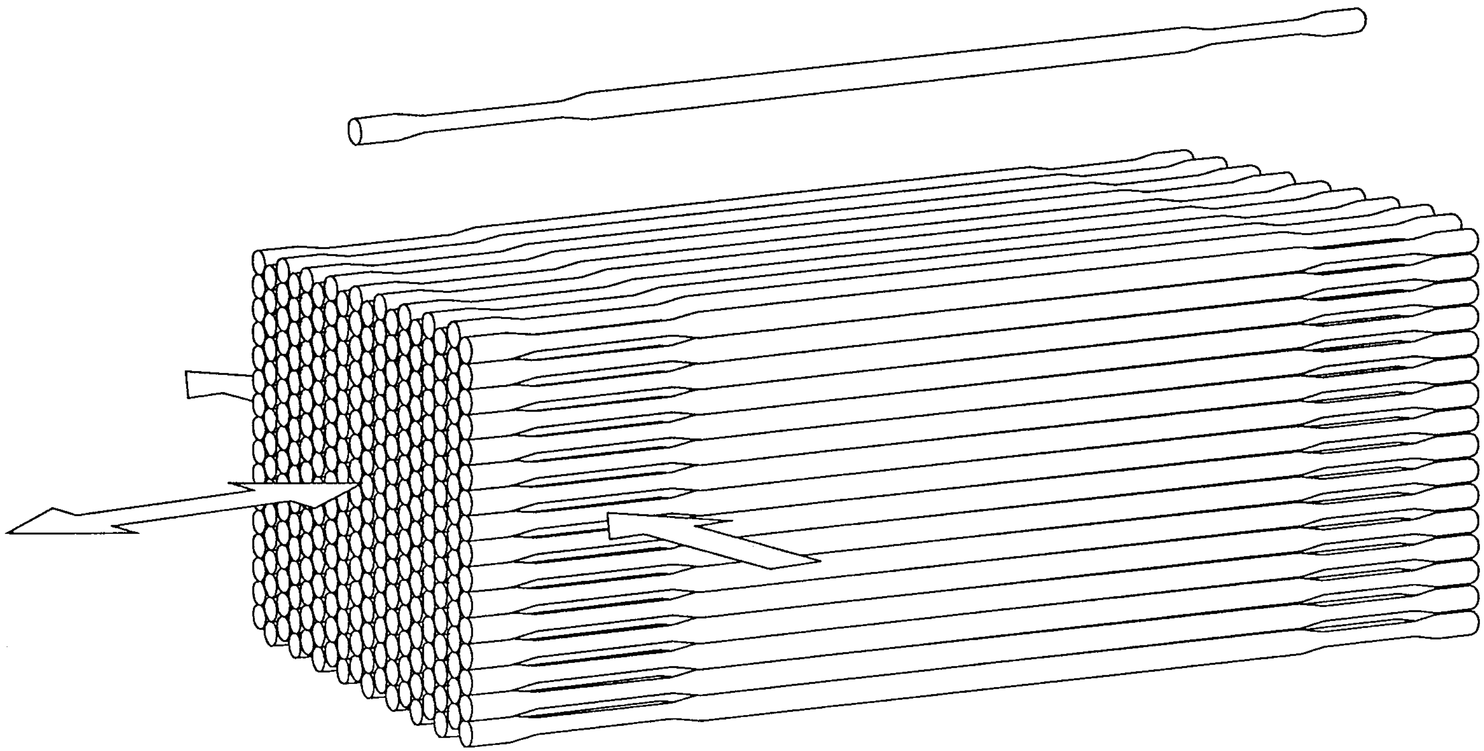


Fig. 3

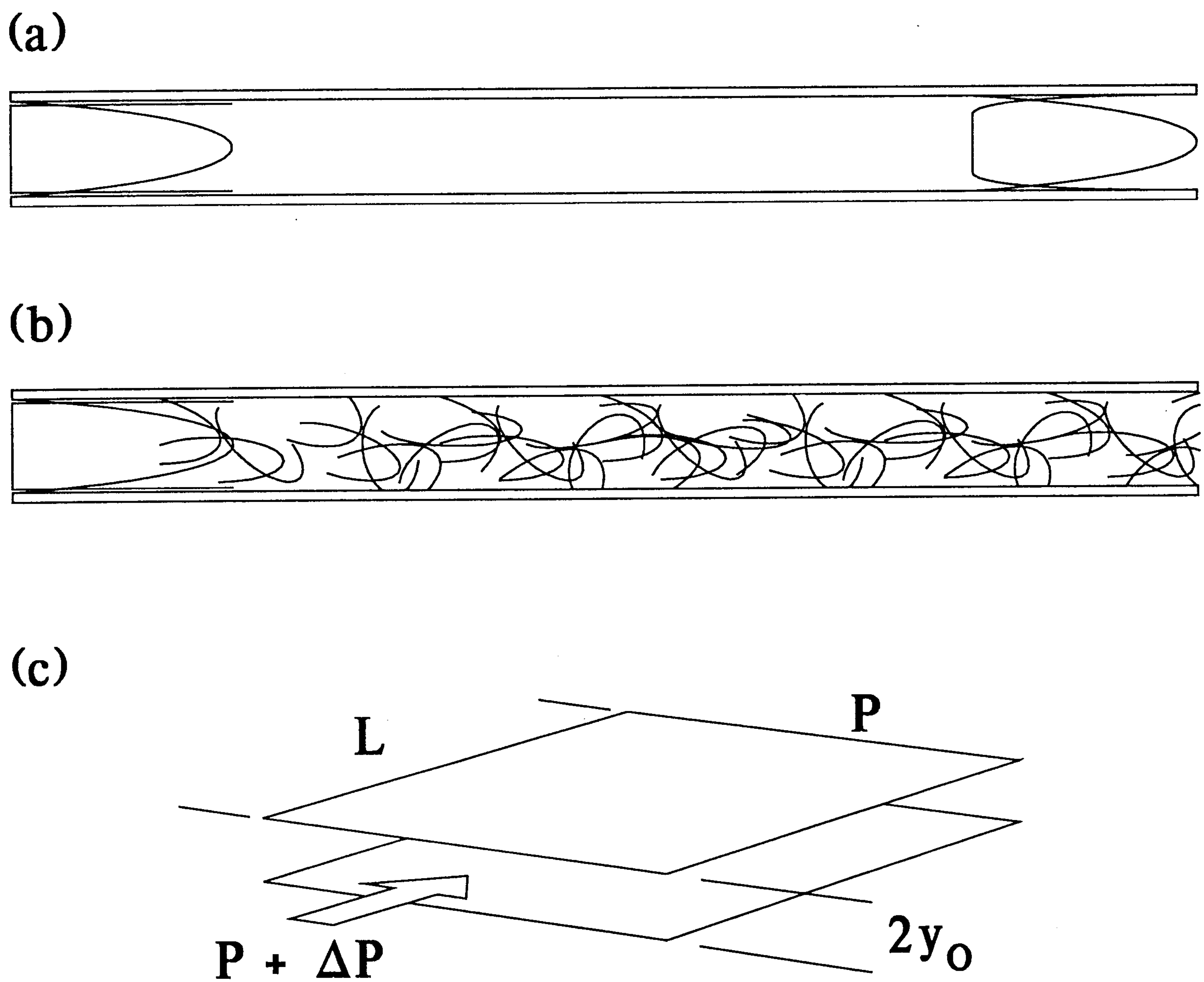



Fig. 4

## APPENDIX B

### Patent Disclosure for a High Power Oscillatory Drive

While most applications of oscillatory drives require modest power levels (e.g. less than 1 kilowatt in acoustic applications), there are some applications which require high power levels (e.g. many KW) and large displacements (on the order of centimeters, or acoustic Mach numbers as large as 0.1). As an example, one application would be for a large thermoacoustic refrigerator. High power, large displacement drives may be made with cranks, cams, unbalanced loads, etc. Such drives are usually limited to relatively low frequencies; for oscillatory motion, acceleration and force are proportional to frequency squared, with the result that higher frequencies place excessive loads on crank or cam bearings. High power drives for higher frequencies are typically electromechanical "shakers", which employ coils of wire positioned, with compliant suspensions, in high magnetic fields. High power densities are achieved by establishing the static magnetic field with permanent magnets made from state-of-the-art ferromagnetic materials. These devices are limited by the critical Joule power dissipation of the wire in the moving coil, above which the wire fuses. This problem is particularly restrictive in oscillatory devices, because in this case the coil spends much of the time at or nearly at rest, and there is no electromotive force (emf) induced back into the coil to limit Joule power dissipation and heating. In an electric motor, the armature coils are constantly in motion, and the induced back-emf prevents catastrophic Joule heating in the coil.

In this patent disclosure it is proposed that oscillatory drives be built using the rotary motion of motors, and converting the rotary motion to oscillatory motion using only permanent magnets. For an electromechanical drive, electric motors would be used, and since the coils would be in constant motion, the problem of coil wire fusion would be alleviated. In general, the non-contact nature of forces between permanent magnets in converting rotary motion to oscillatory motion would circumvent problems of wear for crank bearings, cams, etc. The method would permit high drive powers because of the high energy densities available with state-of-the-art permanent magnets. Readily available magnets have energy densities of $2.5\text{x}10^5$ joule/m$^3$. The force between two 10 cm diameter magnets would be on the order of 2000 N. If this force were used to drive an oscillator with a 1 cm displacement amplitude at 60 Hz, then the power delivered would be several kilowatts. Larger magnets, multiple sets of magnets, and higher frequencies would increase the available power proportionally, and drive powers of many kilowatts would be possible. Multiple sets of magnets could also be used to increase the frequency of oscillation for a given rotation speed of the motor. An example is described below.

A particular model for converting rotary motion to oscillatory motion using permanent magnets is illustrated in Fig. 1. The model consists of three parallel disks, each free to rotate independently on a common shaft. The middle disk is connected to springs which allow the disk to undergo torsional oscillations. The outer disks are driven by motors (or a motor and gears) in rotary motion at the same speed but in opposite directions. Each disk contains a set of magnets which are oriented so as to attract the magnets in the neighboring disk. By having N magnets equally spaced around each disk, the torsional

drag between the disks is increased and the frequency of oscillation of the middle disk is N times the rotation speed of the outer disks.

The operation of the model in Fig. 1 is depicted in Fig. 2. The operation is interesting because the power delivered to the oscillating disk is a nonlinear function of its amplitude squared, as it would have been for the customary driven oscillator. Fig. 2a and 2c show angular displacements for a magnet in the oscillating middle disk (solid line) and the passages of magnets in the counter-rotating disks (dashed lines) as functions of time. Fig. 2b and 2d show the resulting force on the magnet in the middle disk as a function of time. A small amplitude case is illustrated in Fig. 2a and 2b, and a large amplitude case is illustrated in Fig. 2c and 2d.

As a rotating magnet passes the oscillating magnet in the small amplitude case (as illustrated in Fig. 2a), a force is first felt in one direction, and then as the rotating magnet passes, a force is felt in the opposite direction. This is illustrated by the passage at the center of Fig. 1a and by the bipolar peaks in the center of Fig. 2b; the counter-rotating magnet would produce the bipolar peaks at the left and right in Fig. 2b. I might be thought that the bipolar peaks would cancel, resulting in no net driving force. However, the phasing of the disks may be established so that the velocity of the oscillating disk also reverses with the force, so that the power delivered is nominally positive. However, with this phasing the maximum speed of the oscillating disk, indicated by the arrows in Fig. 1b, occurs near a minimum in the force, with the result that a relatively small amount of power is delivered to the oscillating disk.

In the large amplitude case, the amplitude and phase may be set so that the rotating magnets pass the oscillating magnet and then travel together with the oscillating magnet for some fraction of the cycle, as illustrated in Fig. 2c. This results in a force which extends through the points of maximum velocity, as shown near the arrows in Fig. 2d, and produces a relatively large amount of power delivered to the oscillating disk. The amplitude (approximately $1/2\pi$times the spacing of the rotating magnets) and phasing illustrated in Fig. 2c is optimal, and produces more than twice the power (normalized with the amplitude squared) delivered by the situation of Fig. 1a.

A version of the model described above has been constructed and tested. The test device used small cylindrical rare earth cobalt magnets of diameter 2 cm and thickness 0.6 cm, each weighing 15 gm. Because of modest tolerances in the test device, the gap between the disks was relatively large, 0.2 cm, and the resulting torsional drag between the disks was only $\sim$70 N. At the resonance frequency of the middle disk (30 Hz) the amplitude of oscillation was 0.5 cm. By measuring the quality factor of the oscillator (using the free decay of the middle disk oscillator), the power delivered to the oscillator was determined to be 33 W, in good agreement with the predicted value.

It may be noted that for the model illustrated in Fig. 1, the rotating magnets also produce an unwanted oscillatory torque transverse to the shaft. This effect may be eliminated by using a hollow middle oscillating disk with a rotating disk inside, and outer disks rotating

together, but opposite to the inside disk. In this configuration the forces parallel to the shaft cancel, and the torque transverse to the shaft is eliminated.

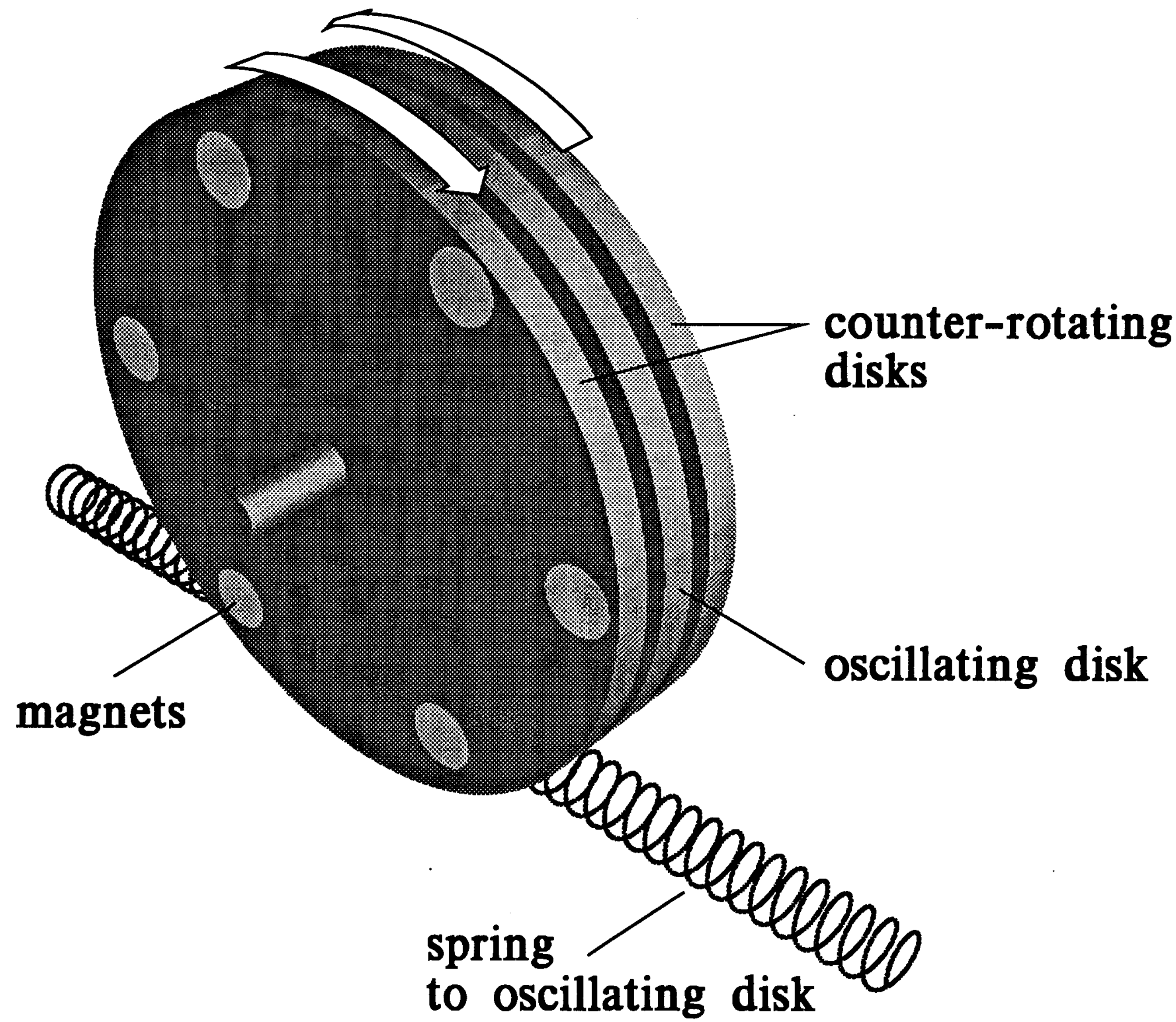
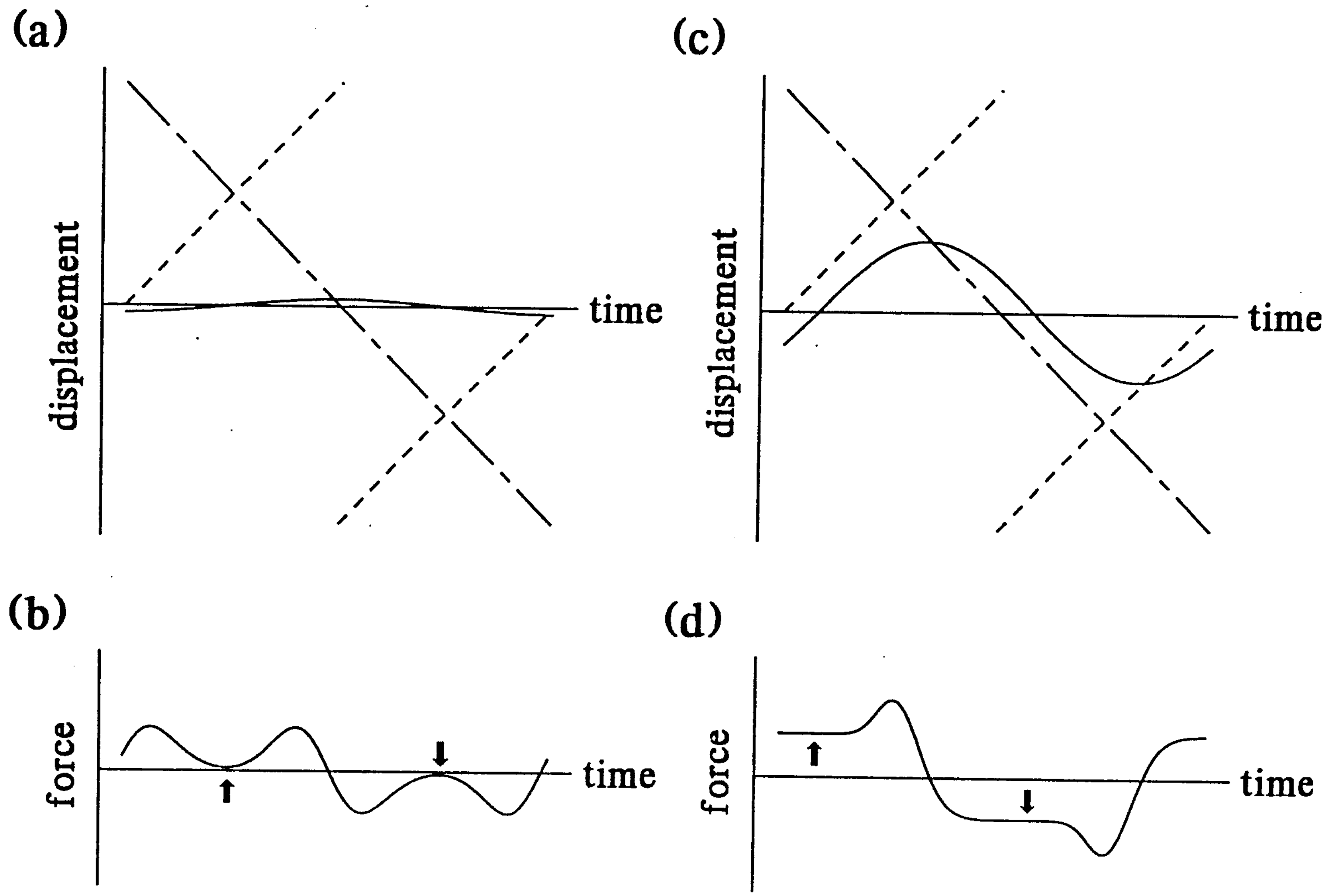

counter-rotating
disks
oscillating disk
magnets
spring
to oscillating disk

Figure 1.



(a)
displacement
time
(c)
displacement
time
(b)
force
time
(d)
force
time


Figure 2.